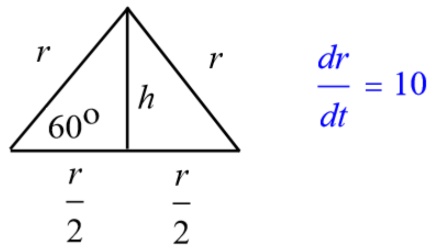
Exercise 31

The sides of an equilateral triangle are increasing at a rate of 10 cm/min. At what rate is the area of the triangle increasing when the sides are 30 cm long?

Solution

At any time, the sides of an equilateral triangle all have the same length r.



The area of this triangle is

$$A = \frac{1}{2}(r)(h).$$

Since we want to know dA/dt when $r = 30$, use trigonometry to eliminate h in favor of r.

$$\sin 60^\circ = \frac{h}{r} \quad \rightarrow \quad \frac{\sqrt{3}}{2} = \frac{h}{r} \quad \rightarrow \quad h = \frac{\sqrt{3}}{2}r$$

As a result, the area becomes

$$A = \frac{1}{2}(r)\left(\frac{\sqrt{3}}{2}r\right)$$

$$= \frac{\sqrt{3}}{4}r^2.$$

Take the derivative of both sides with respect to time by using the chain rule.

$$\frac{d}{dt}(A) = \frac{d}{dt}\left(\frac{\sqrt{3}}{4}r^2\right)$$

$$\frac{dA}{dt} = \frac{\sqrt{3}}{4}(2r) \cdot \frac{dr}{dt}$$

$$= \frac{\sqrt{3}}{2}r \cdot (10)$$

$$= 5\sqrt{3}\,r$$

Therefore, the rate at which the area is changing when the sides are 30 cm long is

$$\left.\frac{dA}{dt}\right|_{r=30} = 5\sqrt{3}\,(30) = 150\sqrt{3}\,\frac{\text{cm}^2}{\text{min}} \approx 259.808\,\frac{\text{cm}^2}{\text{min}}.$$